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Journal of Sound and Vibration 292 (2006) 777–787
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Analytical model of a slotted bolted connection element and its behaviour under dynamic load

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Received 14 May 2005; received in revised form 11 August 2005; accepted 4 September 2005
Available online 1 December 2005

Abstract

Slotted bolted connection elements (SBCEs) have been used extensively in steel frame structures to mitigate the dynamic response by providing hysteretic damping and nonlinear stiffness to the structure. But an analytical model on this energy dissipation mechanism is absent. This paper presents an analytical model to describe the shear force–deformation behaviour of such an element with two sliding steel plates and a single frictional interface. The initial stiffness and other model parameters can be adjusted within a relatively wide range to control its energy dissipation property. The SBCE is included into a single-degree-of-freedom steel frame, and the behaviour and effect of the structure are studied with cyclic and seismic base excitation. The SBCE is shown to perform effectively in vibration mitigation under the cyclic and earthquake loadings.

1. Introduction

The heavy losses caused by strong earthquakes have led to a strong need for mechanical energy dissipating devices to mitigate the effects of seismic forces in civil structures. Techniques that are effective to mitigate the effects of seismic forces on structures, but without increasing the structural strength and rigidity, are of great value to structural vibration control. Although active and semi-active approaches for structural control have been extensively investigated in the past [1], passive control systems are still prevailing in engineering practice due to their simplicity and the low cost of installation and maintenance.

A common feature of all passive control systems is the ability to reduce the dynamic response of a structure by dissipating the vibration energy together with a shift in the natural frequency of the structure [2]. A structure will not collapse in general when the energy dissipation capacity is designed larger than the input energy. There are three primary sources of damping in a structure to achieve this aim. These are the viscous damping from structural and non-structural components, the hysteretic damping at plastic hinges and hysteretic damping arose from friction at the sliding interfaces.

The slotted bolted connection element (SBCE) incorporated at intersections or joints of a steel frame is allowed for in design standards [3]. An SBCE is a semi-rigid bolted connection with nonlinear relationship

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0022-460X/$ - see front matter
doi:10.1016/j.jsv.2005.09.028

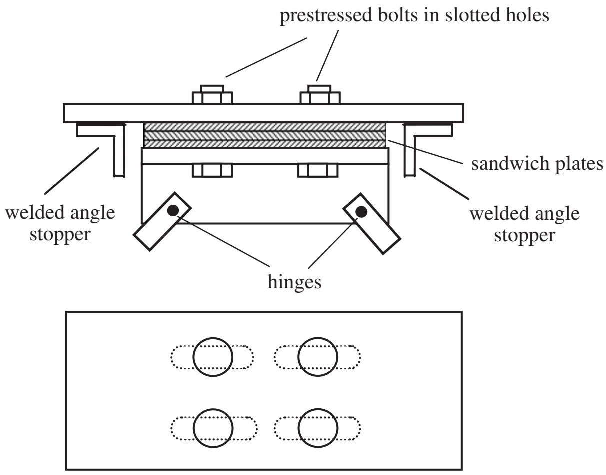

Fig. 1. Configuration of SBCE.

between the shear force and the relative slippage at the interfaces. Basically, high-strength friction grip bolts are used in an SBCE to connect two parts of the steel frame with slotted holes in the components, and the frictional interface may consist of sandwiched plates as shown in Fig. 1. Prestressed force is applied on the interface in the normal direction to provide the SBCE with sufficient initial stiffness before significant slip occurs. It behaves nearly rigidly when subject to small in plane shear force. Large slippage may occur between the surfaces of the element when under severe excitation, and energy is dissipated with the accompanying hysteretic damping. To protect the prestressed bolts from transverse shear, the maximum relative displacement is limited by a stopper as shown in Fig. 1. It is noted that the summation of the gaps between the stoppers and the edges of the element should be smaller than the length of the slotted hole minus the bolt diameter.

Basic analytical and experimental works on the SBCE have been published previously. Tritchkov et al. [2] have investigated an adjustable slippage element to control the structural vibration by considering the nonlinear variation of the element stiffness. An analytical method has been proposed to analyse the structure with the SBCEs. Grigorian et al. [4] have tested the behaviour of two types of SBCEs, in which the friction surfaces are lined with steel or brass pads. Popov et al. [5] also included the SBCEs in an experimental model in a shaking table test, and they were found very effective in dissipating most of the input energy. However the work was purely based on experiments and no theoretical model on the device has been proposed. A structure with SBCEs needs an analytical model to accurately predict the responses of the structural system under dynamic loading for design purposes.

This paper presents the development of the SBCE model and to study the behaviour of a steel frame including the SBCE under dynamic loads. An exponential model on the distribution of the contact surface peak height is used to describe the force–deformation behaviour of the SBCE. The parameters of the element, include the initial stiffness, maximum frictional force and slippage limit, are related to the normal compression force and its distribution, material type and the number of sandwiched plates, geometry and surface characteristics. The shape of the force–displacement curve of the element can be controlled by adjusting these physical parameters within a relatively wide range. The behaviour and effectiveness of the proposed model in vibration mitigation are investigated by considering a single-degree-of-freedom (sdof) steel frame subjected to cyclic loading and earthquake excitation.

2. Behaviour of an SBCE under cyclic loading

The monotonic force–deformation relation in the tangential direction of the frictional joints is first defined and then extended to the modelling of the cyclic behaviour based on Masing's rule [6] as follows:

The most popular existing model to describe the force–deformation relationship in a general frictional slip interface is the bilinear model [7], which has the disadvantage of slope discontinuity, and this is undesirable in the numerical analysis and modelling. With the assumption of an exponential distribution of peak height of spherical contact elements, Shoukry [8] developed a microslip element to model the frictional behaviour between two metallic interfaces, by using Mindlin's spherical contact element [9] as the basic element. The force–deformation relation of the frictional interface is given by

$$Q = \mu N\left[1 - \exp\left(\frac{-\gamma}{\sigma}v_c\right)\right], \tag{1}$$

where Q and v_c are the force and lateral deformation of the interface, μ, N are the friction coefficient and normal force acting on the interface respectively, σ is the standard deviation of the peak height distribution of the contact element, and γ is a constant equal to $2(1-v)/[\mu(2-v)]$, where v is the Poisson ratio.

With consideration of the usual configuration of a slotted bolted connection in a structural frame, a mathematical expression can be written for the load–deformation relation of a frictional joint in the tangential direction as

$$\begin{aligned} Q &= Q_s\left[1 - \exp\left(-\frac{K_0}{Q_s}v_c\right)\right] \quad \text{if } v_c \leqslant v_{cl}, \\ Q &= Q_s\left[1 - \exp\left(-\frac{K_0}{Q_s}v_{cl}\right)\right] + k_l(v_c - v_{cl}) \quad \text{if } v_c > v_{cl}, \end{aligned} \tag{2}$$

where v_{cl} is the slippage threshold of the bolted hole, Q_s is the friction limit which is equivalent to μN in Shoukry's expression, K_0 is the initial stiffness, i.e. $K_0 = \mathrm{d}Q/\mathrm{d}v_c|_{v_c=0}$, which is equivalent to $\mu N\gamma/\sigma$ in Shoukry's expression, and k_l is the contact stiffness after the slippage threshold is reached. These properties are shown in Fig. 2(a) where Point B is the point defining the slippage threshold. It can be seen that the stiffness of joint changes nonlinearly with the magnitude of shear loading. The joint behaves rigidly under small loading, and it softens at the application of severe loading. However when the slip threshold is reached, the bolt is in contact with the edge of the bolted hole, and the stiffness of joint is governed by the constant stiffness k_l.

Eq. (2) above is physically important because the joint parameters in the loading curve, such as the initial stiffness and the frictional resistance force, are related to other physical parameters, like the normal pressure and friction coefficient. It is noted that the final frictional force between the two surfaces is important rather than the friction coefficient. Accordingly, the joint properties can be predicted from other physical parameters, and direct testing for the force–deformation characteristics may not be necessary.

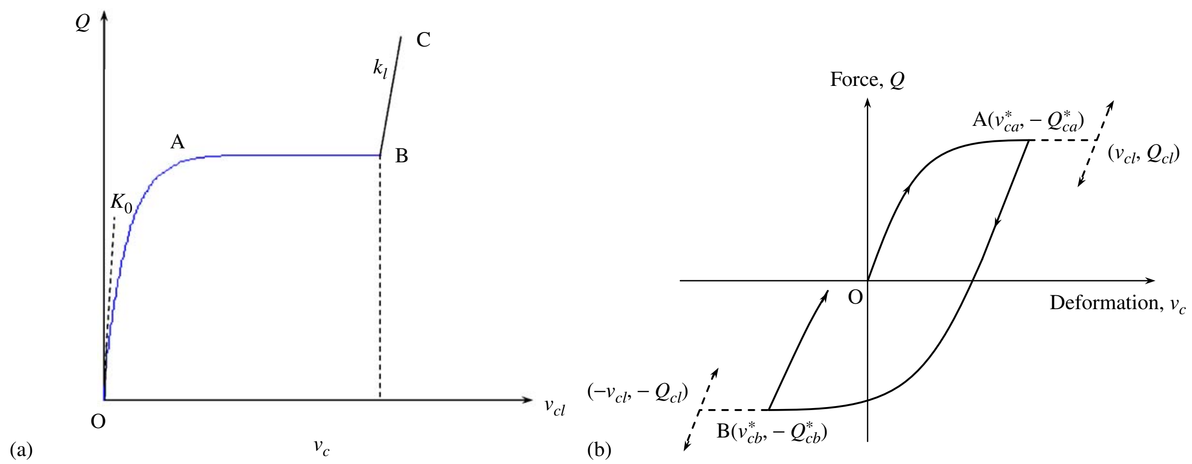

Fig. 2. (a) Virgin loading curve of an SBCE. (b) Hysteretic curve under cyclic loading.

In the following formulation, Eq. (2) is used to represent the virgin loading curve of the frictional joint in the tangential direction. The corresponding instantaneous connection stiffness in the virgin curve is given by

$$k_v = \frac{\mathrm{d}Q}{\mathrm{d}v_c} = K_0 \exp\left[-\frac{K_0}{Q_s} v_c\right] \quad \text{if } |v_c| \leqslant v_{cl},$$
$$k_v = \frac{\mathrm{d}Q}{\mathrm{d}v_c} = k_l \quad \text{if } |v_c| > v_{cl}. \tag{3}$$

It has been observed from cyclic tests that the force–slippage loops of an SBCE are stable and reproducible [4]. Based on these experimental observations, the present study assumes that the static monotonic force–slippage curve can be extended to cyclic and dynamic analysis based on the Masing rule [6]. The unloading and reloading curves within the slippage limit of the SBCE can be described from Eq. (2) as follows:

$$\left|\frac{Q - Q^*}{2}\right| = Q_s\left[1 - \exp\left(-\frac{K_0}{Q_s}\left|\frac{v_c - v_c^*}{2}\right|\right)\right] \quad \text{if } |v_c| \leqslant v_{cl},$$
$$Q = Q^* + k_l(v_c - v_c^*) \quad \text{if } |v_c| > v_{cl}, \tag{4}$$

where (v_c^*, Q^*) is the point at which load reversal occurs in the range of $|v_c| \leqslant v_{cl}$. When $|v_c| > v_{cl}$, the loading and reloading process is on a straightline with slope k_l. Therefore, the corresponding instantaneous connection stiffness of both the unloading and reloading curves can be expressed as

$$k_v = \frac{\mathrm{d}Q}{\mathrm{d}v_c} = K_0 \exp\left(-\frac{K_0}{Q_s}\left|\frac{v_c - v_c^*}{2}\right|\right) \quad \text{if } |v_c| \leqslant v_{cl},$$
$$k_v = \frac{\mathrm{d}Q}{\mathrm{d}v_c} = k_l \quad \text{if } |v_c| > v_{cl}, \tag{5}$$

in which, $v_c^* = v_{ca}^*$ when the slippage deformation happens after the reversal point A in Fig. 2(b). If point B is the next reversal point, the reloading stiffness is obtained by replacing v_{ca}^* with v_{cb}^* in Eq. (5).

3. Behaviour of a sdof system incorporating a single SBCE

The effectiveness of the proposed SBCE model is investigated by comparing the response of a simple one-storey braced frame as shown in Fig. 3(a) to the response of a similar frame equipped with an SBCE as shown in Fig. 3(b). The hinge in the middle in Fig. 3(a) denotes the hinge action between the bracing members and the horizontal beam which is continuous at this point. It is assumed that the structural deformations do not change the normal reaction at the SBCE. The lateral stiffness of the reference structure in Fig. 3(a) is constant for small displacement and its force–displacement curve may be characterized by the slope k_{r0} in Fig. 3(c). Adding an SBCE to the frame will generally soften the structure slightly because of the additional flexibility. The lateral initial stiffness of the frame is characterized by the slope k_0 as shown in Fig. 3(c). The larger the normal prestressing force, the closer the value of k_0 to k_{r0}. Also when the excitation is small, the slope of the force–displacement curve will remain relatively unchanged and the dynamic response of the structure will be very close to that of the reference structure. The relative slippage may be eliminated if the prestressed clamping force is infinite. In this case, the connection element of the frame is rigid, and the total stiffness of the frame would be equal to the stiffness of reference frame.

The modified frame is modelled as an sdof system consisting of a mass, an SBCE, and a dashpot, as shown in Fig. 3(d). The total lateral stiffness of the frame includes the stiffness of the SBCE, the bracing elements and that of the frame, and is a nonlinear function of both the displacement and velocity. Since the stiffness of the SBCE and the frame are in series, the combined stiffness of the system can be expressed as

$$\begin{aligned} k(x,\dot{x}) &= \frac{k_{r0}k_v(x)}{k_{r0} + k_v(x)} \\ &= \frac{k_{r0}}{k_{r0}/k_v(x) + 1} \\ &= \frac{k_v}{1 + k_v/k_{r0}(x)}, \end{aligned} \tag{6}$$

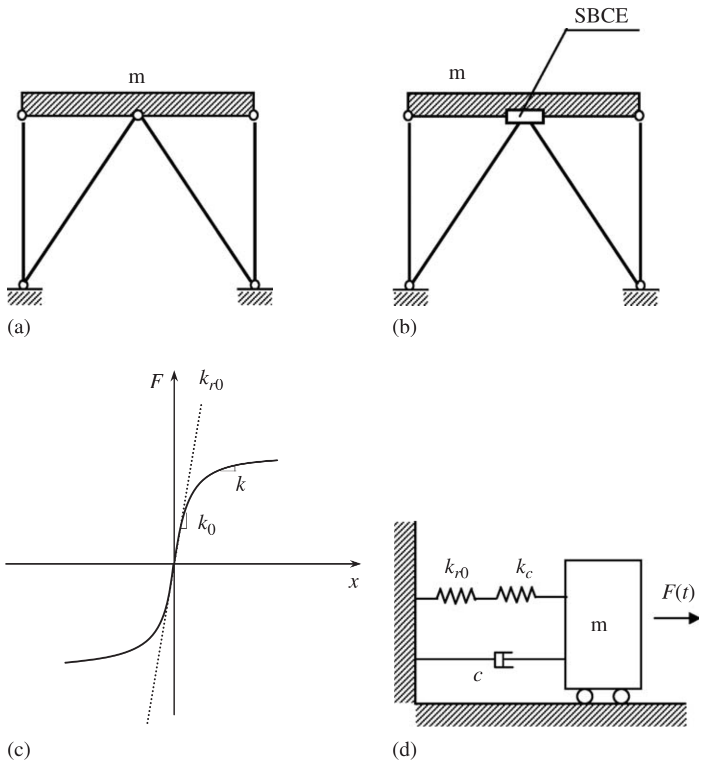

Fig. 3. Application of SBCE to sdof system. (a) Reference braced frame; (b) braced frame incorporating the SBCE; (c) force–displacement curves of the frames; (d) model of the sdof system equipped with SBCE.

where k_{r0} is the lateral stiffness of the reference structure, and k_v is the lateral stiffness of the SBCE which may be found from Eq. (5). The relative slip between the interfaces of an SBCE can be written as

$$\Delta v_c = \frac{k_v}{k_{r0} + k_c(x)} \Delta x. \tag{7}$$

The differential equation governing the motion of the system may be written as

$$m\ddot{x} + c\dot{x} + k(x, \dot{x})x = F(t) \tag{8}$$

in which $F(t)$ is the external excitation, m is the mass of the system, c is the viscous damping constant of the system, and k is the combined tangential stiffness of the structural system. In the subsequent nonlinear analysis process, equilibrium equation at time $t + \Delta t$ can be sought on the basis of the last known equilibrium state at time t [10]. The incremental equilibrium equation can be written as

$$m\Delta\ddot{x} + c\Delta\dot{x} + k(x, \dot{x})\Delta x = \Delta F(t) \tag{9}$$

in which ΔF is the incremental applied force; and Δx, $\Delta\dot{x}$ and $\Delta\ddot{x}$ are the incremental displacement, velocity and acceleration, respectively.

For a linear oscillator, only the frequency content of the excitation force affects the response amplitude of the system (together with the system's damping). The rate of change of the excitation frequency for a nonlinear oscillator also affects the system response. Hence for the harmonic excitation with a gradually changing frequency, the frequency–amplitude relationship of a sdof nonlinear oscillator may be discontinuous. It would exhibit bifurcation at certain values of stiffness and damping parameter [11]. Newmark method is used for the dynamic response analysis of the frame under external excitation.

There are two major parameters in the SBCE model that affect the dynamic responses. They are the initial stiffness ratio, $r = K_0/k_{r0}$, and the slippage load ratio, $l_f = Q_s/F_0$, where F_0 is the amplitude of the applied harmonic load. An infinite prestressing force corresponds to a slippage load ratio $l_f = \infty$ and to a linear oscillator with stiffness $k = k_{r0}$. For both the cases of $l_f = 0$ and ∞, the undamped response due to harmonic excitation is unbound because both cases represent a linear elastic oscillator in resonant. Thus, there should be a value of the slippage threshold (or the prestress level) for which the response ratio $x_{\max}/x_{st}$ is a minimum, where $x_{\max}$ is the maximum displacement response of the oscillator and $x_{st} = F_0/k_{r0}$ is the displacement under the static action of the force. Numerical studies conducted below show that such a value exists and prior knowledge of this information before construction allows the engineer to design for the best dynamic performance of the structure.

An equivalent sdof system is used for the solution of Eq. (9) by dividing with the mass m and assuming m takes up a specified value, say unity. The equation is then solved by Newmark integration.

4. Case studies

4.1. Under harmonic excitation

The natural frequency of the reference frame is selected to be 14 rad/s and it has a damping ratio of 0.02. The response ratio $x_{\max}/x_{st}$ under harmonic excitation nearly equals to $1/2\zeta$ which is 25.

The SBCE has a slippage limit of ± 5.0 mm. The response ratio of the frame with the SBCE resulting from a harmonic excitation at the natural frequency of the frame is shown in Fig. 4. The response ratio is shown always smaller than 25 when different parameter combinations are selected. This means that there is always some advantage in vibration mitigation when SBCE is used. When either r or l_f is increased, the response ratio $x_{\max}/x_{st}$ increases with a limiting value of 25.

When the initial stiffness ratio changes, the optimal value of l_f, corresponding to a minimum response ratio, ranges from $l_f = 1.6$ for $r = 0.5$ to $l_f = 0.5$ for $r = 10$ as shown in Fig. 4. However the value of r and l_f cannot be taken very close to zero with confidence because a small variation from the desired value would result in a large jump in the response ratio. Also technically, it is impossible to have a SBCE with an initial stiffness close to zero. Therefore l_f could be selected in the range equal to or larger than the optimal value in practice.

If the physical properties of the reference frame is changed, and the exciting frequency changes to match the natural frequency of the reference frame, the response ratio changes as shown in Fig. 5. The reduction in the vibration peak amplitude is very significant over a very wide range of natural frequency or stiffness of the frame, and we have an optimum value in the range of 8–60 rad/s to have only 10% of the maximum response ratio of the reference structure, i.e. 2.5. There is only a small variation in the response ratio for different combinations of K_0 and l_f over this range. The response will be larger when the natural frequency is either smaller or larger than the above range.

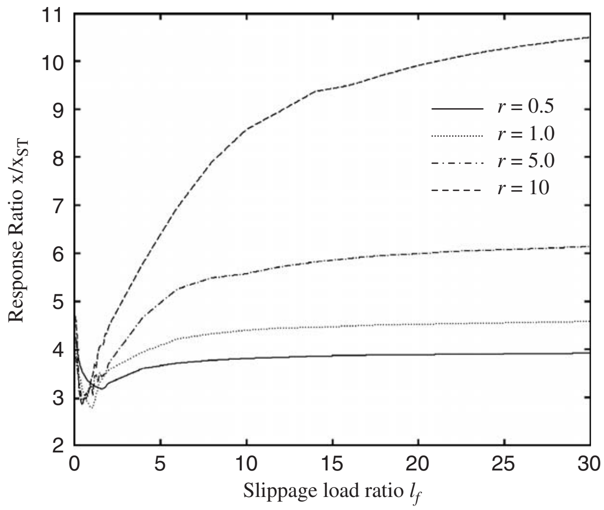

Fig. 4. Response amplitude curves at different initial stiffness ratio and slippage load ratio.

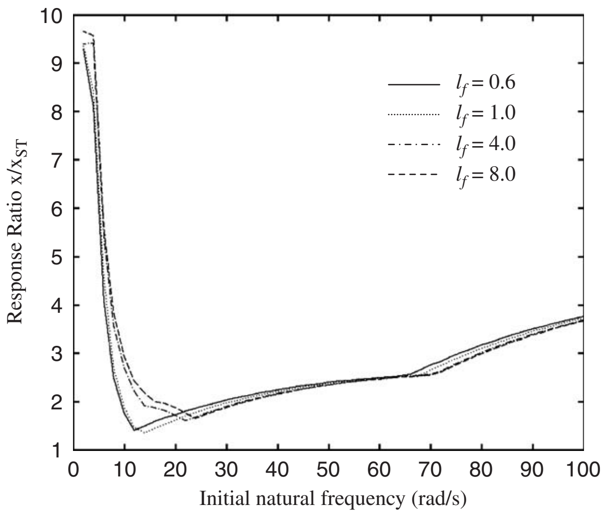

Fig. 5. Response amplitude curves at different natural frequency.

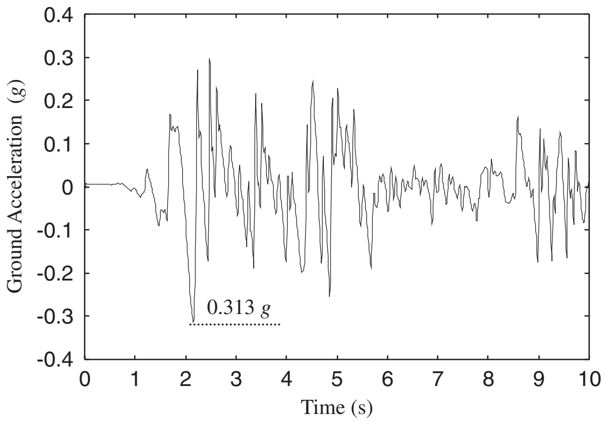

Fig. 6. Accelerogram for NS component of El Centro (1940) earthquake.

4.2. Under ground excitation

The effect of the system parameters is further investigated by subjecting the sdof system to a ground excitation loading corresponding to that in a real earthquake. The record of the 1940 El Centro N–S earthquake component motion is used in this study. The first 10 s record is shown in Fig. 6 with a peak ground acceleration of 0.313 g.

Pseudo-acceleration response spectra is a popular tool to assess the seismic properties of civil structures [12] because it gives information over a wide range of frequency. The nonlinear pseudo-acceleration response spectra of the 1940 El Centro earthquake ground motion are developed numerically for different SBCE parameters and they are shown in Figs. 7 and 8. The nonlinear spectra are calculated for the system with the system damping factor $\zeta = 0.005$. The graphs are plotted for the values of initial stiffness ratio $r = 1$ and 5, and with the slippage load ratio $l_f = 1.0$, 4.0, 8.0 and ∞ where $l_f = \infty$, corresponds to the reference structure with no SBCE attached. It is noted that the initial stiffness ratio $r = 5$ associates with a slight shift of the circular frequency of the system from 14 to 12.78 rad/s. As might be expected, the most significant effect of the element is in the reduction of the local peaks of the response spectra within the range of initial natural periods shown, thus reducing the ultimate response of the frame to the ground acceleration.

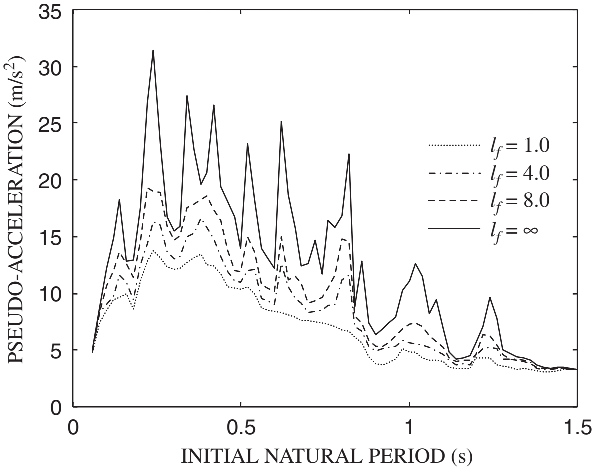

Fig. 7. Nonlinear response spectrum from El Centro ground motion; $\zeta = 0.005$; $r = 1.0$.

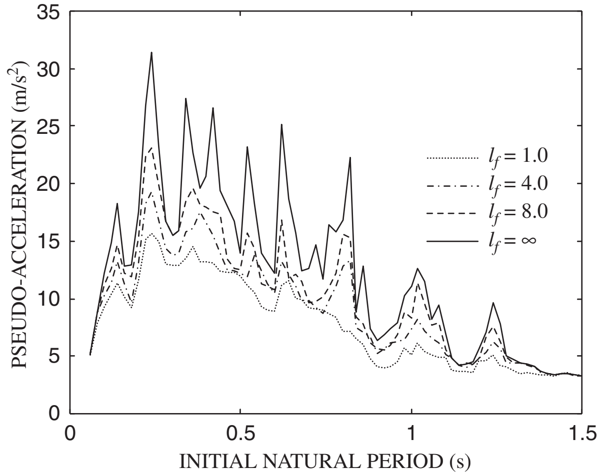

Fig. 8. Nonlinear response spectrum from El Centro ground motion; $\zeta = 0.005$; $r = 5.0$.

The effects of the parameters of the SBCE is further studied by calculating the nonlinear pseudo-acceleration response spectra of the 1976 N–S component of the ground acceleration at the site in Hongshan, China of the Tangshan earthquake, for different SBCE parameters and they are shown in Figs. 10–13. The acceleration record is shown in Fig. 9 with a peak acceleration of $-7.24\,\mathrm{cm/s^2}$. The nonlinear spectra are calculated for the system with the system damping factor $\zeta = 0.0$ or 0.015. The graphs are plotted for a particular value of the initial stiffness ratio $r = 1$ or 5, and for a value of the slippage load ratio $l_f = 1.0$, 4.0, 8.0 and ∞ (see also Figs. 10–13).

The range of the initial natural period, where the softening nonlinearity of the SBCE can effectively reduce the response, varies with both the values or r and l_f. For example, for the case with a low value of the stiffness ratio $r = 1$ and a zero damping factor (Fig. 10), the SBCE is effective for the oscillator over the whole range of initial natural period shown. When the parameter l_f is far from zero and not smaller than their optimum values, the lower their value, the larger will be the reduction in the response. When the frame has a damping factor of $\zeta = 0.015$ and with $r = 1$, the amplitude of the response spectrum shown in Fig. 12 is much smaller than that from zero damping. And the vibration reduction by the inclusion of the SBCE is marginal for range of initial natural period around 0.8, 1.7 and 2.3 s. However there will still be a significant vibration reduction in the overall response by summation over the responses of all the components in the spectrum.

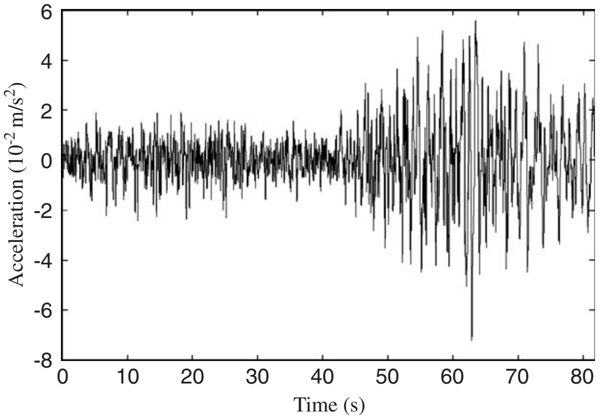

Fig. 9. N–S ground acceleration record in Tangshan earthquake, 1976.

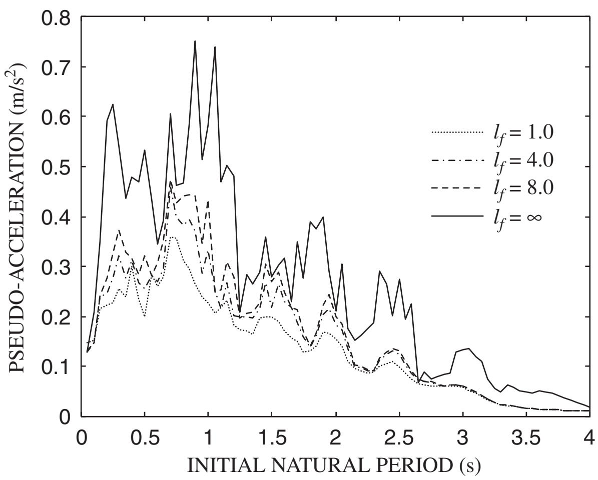

Fig. 10. Nonlinear response spectrum from Tangshan ground motion; $\zeta = 0$; $r = 1$.

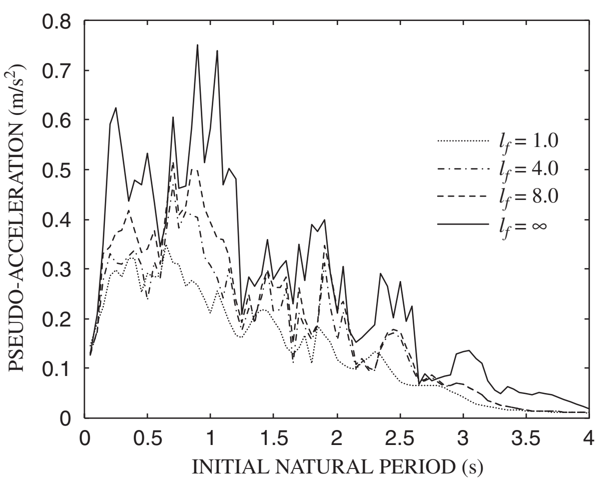

Fig. 11. Nonlinear response spectrum from Tangshan ground motion; $\zeta = 0$; $r = 5$.

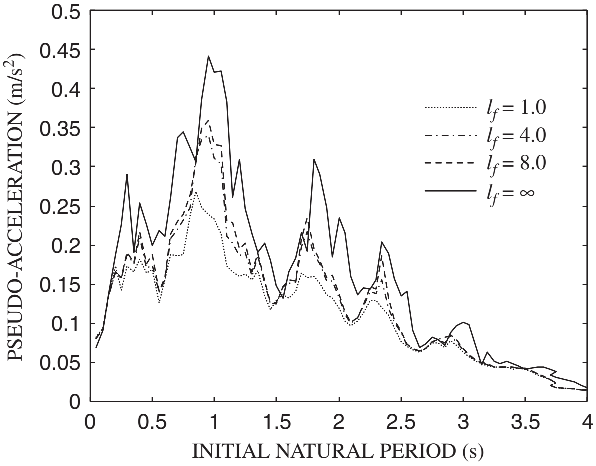

Fig. 12. Nonlinear response spectrum from Tangshan ground motion; $\zeta = 0.015$; $r = 1$.

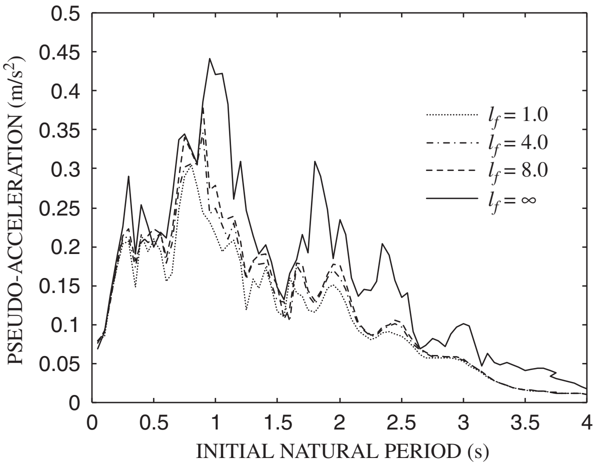

Fig. 13. Nonlinear response spectrum from Tangshan ground motion; $\zeta = 0.015$; $r = 5$.

Despite the above unsatisfactory observation with the Tangshan earthquake, it must be noted that the vibration reduction effect would be different under different seismic input. Therefore the selection of the initial stiffness of the SBCE is not only related to structural behaviour of the frame but also to the frequency components of the acceleration time history of the site where the frame structure will be constructed.

The lowest possible r and l_f values are in practice governed by the structural stiffness requirements for resisting other horizontal loads in terms of the allowable displacement. The effectiveness of the SBCE is associated with a reduction in the force transmissibility and a shift away from the resonance condition due to the additional stiffness from the SBCE. For very high and very low r and l_f values, the effect of the SBCE is similar to that from making a slight change in the natural frequency of the sdof oscillator. The lower the initial stiffness ratio and the slippage load ratio, the greater in general will be the effect of the SBCE, although as discussed earlier, this ratio could not possibly be close to zero.

5. Conclusions

An analytical model is proposed to explain the shear deformation relation of the commonly used slotted bolted connections in steel frame structures. The force–slip deformation relationship is derived based on a microslip element of the frictional interface. The energy dissipation property of the SBCD can be designed for a particular application to reduce the force transmissibility resulting in an enhanced and optimal dynamic performance and a shift in the natural frequency of the structure.

The parameters that can be adjusted include the initial stiffness ratio and the threshold slippage load. They can be adjusted even after the device has been installed in the structure by adjusting the prestress force of the bolts. Although there is a relative flexibility in choosing these parameters in a SBCE, limits are imposed by the requirement of a minimum horizontal stiffness of the whole structure, and by the limit on the magnitude of the prestress forces.

Numerical simulations show that the SBCE is effective in the mitigation of responses for almost all initial stiffness ratios and threshold slippage load under steady-state dynamic load, and an optimal threshold slippage load exists. Pseudo-acceleration nonlinear response spectra from El centro and Tangshan seismic records are calculated and they are compared for different initial stiffness ratio and threshold slippage load. The vibration reduction effect of the SBCE is found significant for most of the initial natural periods of the spectrum.

Acknowledgements

The work described in this paper was supported by a grant from the Hong Kong Research Grant Council Project No. PolyU 5054/99E.

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